Stressing commonalities rather than differences in the teaching of mathematics

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Pimolitel

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4 5
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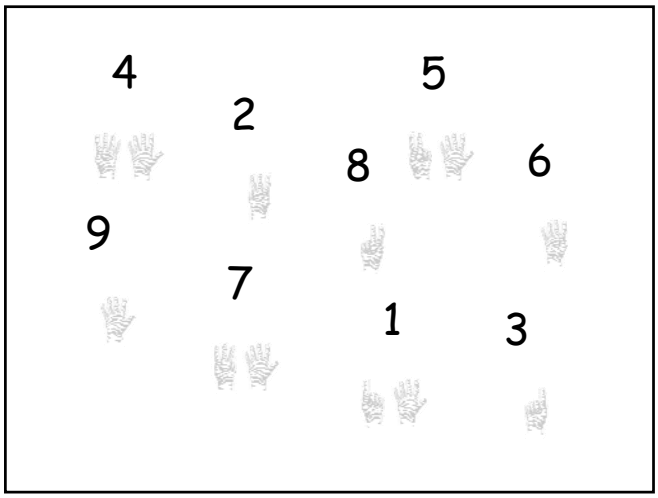

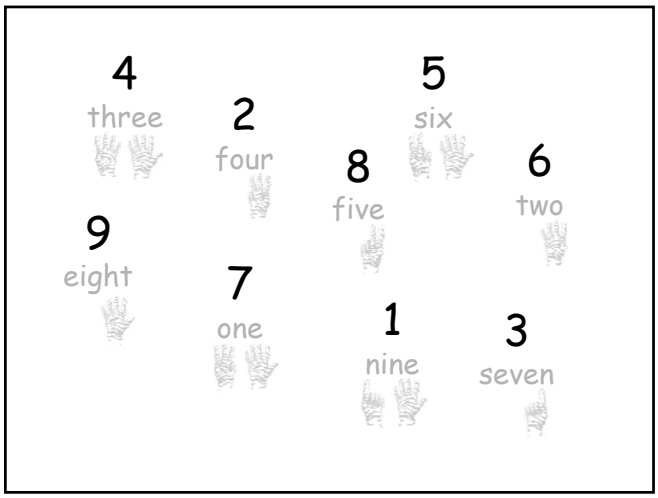

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Some examples of what pre-school children do

- try to do things themselves
- take on challenges which are often in advance of their current skills (indeed they are often bored with doing things they can already do)
- learned a language without anyone explicitly teaching them

Some examples of what ~~pre~~-school children do

- try to do things themselves
- sit back and wait for a teacher to spell everything out
- take on challenges which are often in advance of their current skills (indeed they are often bored with doing things they can already do)
- object if something isn't immediately within their skill level and say "I can't do it"
- learned a language without anyone explicitly teaching them
- fail to learn so much despite being explicitly taught

An activity

Based upon Mason (2008), Joined-up reflections: part 2, *Mathematics Teaching*, 211, pp. 40-43.

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Once you know the perimeter….

- Use the squared paper to draw another shape with a larger perimeter based on the same sized square, where the perimeter could be worked out in a similar way.
- What perimeters can you make?

What did you 'use' in working on this problem?

- *Stressing and ignoring*: certain edges were stressed whilst others were temporarily ignored. In fact stressing and ignoring is involved in the following:
  - *Same/different*: certain edges stopped at the 'same' place as another edge started;
  - *Association*: certain edges were linked with other edges;
  - *Order*: a particular collection of edges were linked together in an order;
  - *Finding patterns, abstracting rules and applying those rules*: initially you might not know how to proceed, then something is noticed, conjectures made, tested, rules established and applied.

Powers of the mind

- These are examples of what Gattegno (1970) called *powers of the mind*. They are an attribute of the mind and are used by everyone: babies, children and adults.

Gattegno, C. (1970), 'Notes on a new epistemology: teaching and education', *Mathematics Teaching*, 50, pp. 2-5.

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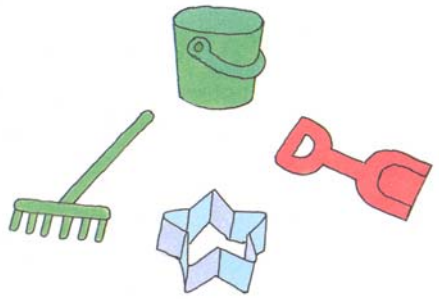
Association



Powers of the mind

- Association

Association

$$a^2 + b^2 = c^2$$

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Association

$P(H) = ½$

Association

Association

$7 + 3$

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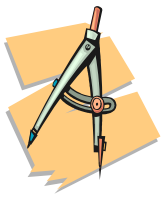
Association

11, 13, 17, 19

Association

(2, 5)

Association



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Powers of the mind

- What other *powers of the* mind are there?
- How did you learn your first language?
- Language is complex
  - The word "chair" is just a sound, how does meaning for this sound become known?
  - Putting words together to make a sentence
  - Rules such as adding "-ed" at end of a verb to get the past tense

The word "chair" is just a sound, how does meaning for this sound become known?	Abstracting **what is the same and what is different** by **stressing and ignoring** aspects of given examples
Putting words together to make a sentence	Sense of **order**
Rules such as adding "-ed" at end of a verb to get the past tense	Noticing **patterns** in examples, establishing **a rule** and **applying that rule to new situations**

Order

One……. two……. three.. four.._____

Once I caught a fish _____

The beginning of counting is no different to learning order in poems, songs, sentences, etc.

Order is fundamental to language and to mathematics.

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Order

- This can be found in:
  - counting;
  - reading algebraic equations;
  - inverse;
  - combinations and permutations;
  - … etc.

Noticing patterns, finding rules and applying rules

- "I goed to the park yesterday."
- "There are sheeps in the field."
- On hearing a bird mimic the word "hello" by saying "hello… hello" a boy (2 years 11 months) said "It said 'hello' two once." **[Perhaps leave this until talking about number names?]**
- 5 year old boy coming back from school saying it is all too hard when he found out that the 'u' sound in 'put' and 'but' is said so differently (seeking rules rather than memorising arbitrary things). **Have the words up one at a time and get them to say them? Then have them together.**

Patterns, finding and applying rules

- This can be found:
  - Nearly everywhere in mathematics!
  - Particularly in algebraic activity.

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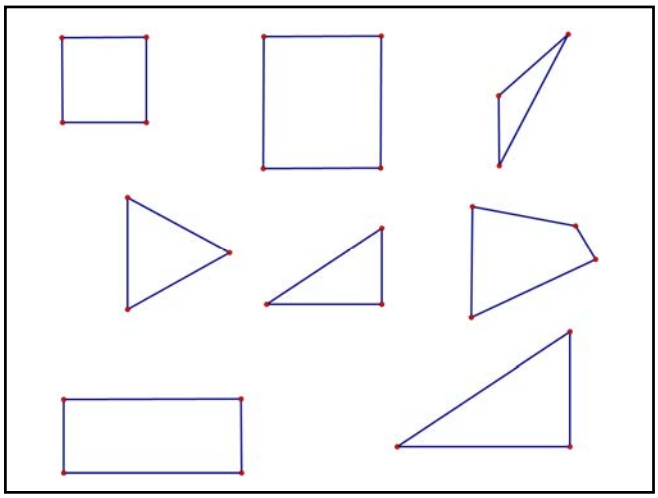
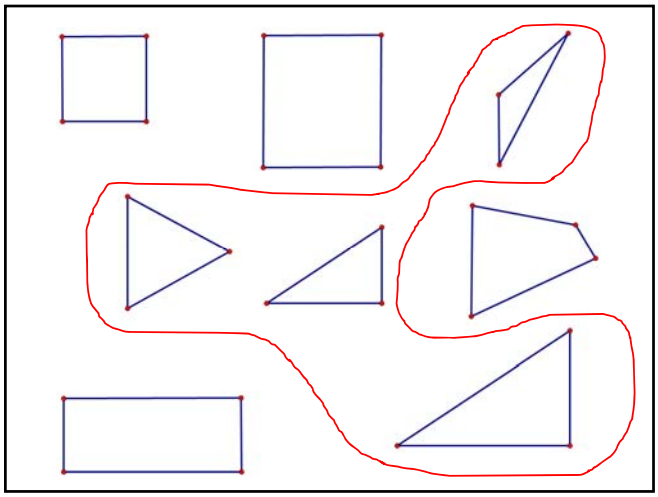
Stressing some aspects ***and ignoring*** others
(leading to what is the ***same*** and what is ***different***)

- **This can be found in the following:**





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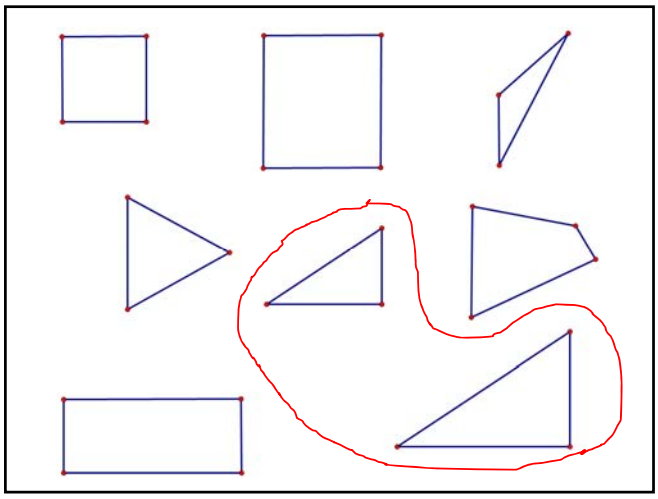

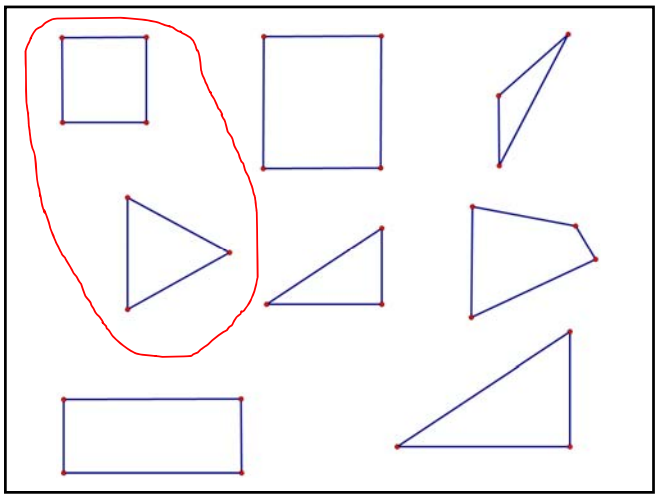

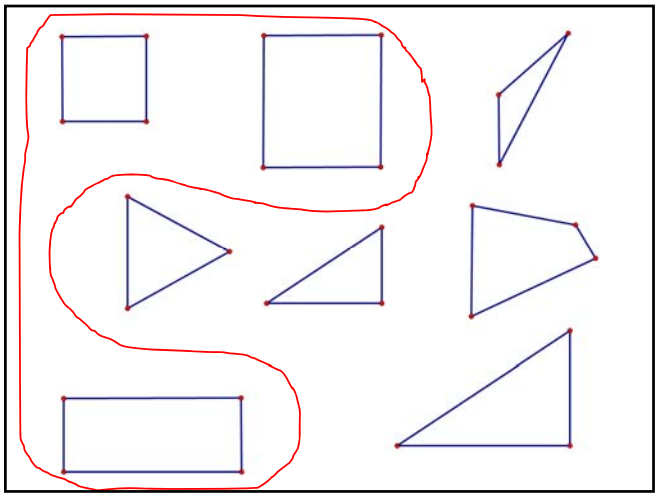

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1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

1	**2**	3	**4**	5	**6**	7	**8**	9	**10**
11	**12**	13	**14**	15	**16**	17	**18**	19	**20**
21	**22**	23	**24**	25	**26**	27	**28**	29	**30**
31	**32**	33	**34**	35	**36**	37	**38**	39	**40**
41	**42**	43	**44**	45	**46**	47	**48**	49	**50**

1	**2**	**3**	4	**5**	6	**7**	8	9	10
11	12	**13**	14	15	16	**17**	18	**19**	20
21	22	**23**	24	25	26	27	28	**29**	30
31	32	33	34	35	36	**37**	38	39	40
41	42	**43**	44	45	46	**47**	48	49	50

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1	2	**3**	4	5	**6**	7	8	9	**10**
11	12	13	14	**15**	16	17	18	19	20
21	22	23	24	25	26	27	**28**	29	30
31	32	33	34	35	**36**	37	38	39	40
41	42	43	44	**45**	46	47	48	49	50

"Two negatives make a positive"?

-3 – -4

-3 + -4

-3 x -4

-3 ÷ -4

-3 ^ -4

$$\int \frac{6x}{3x^2+2}dx$$

$$\int \frac{15x^2+4x-3}{5x^3+2x^2-3x}dx$$

$$\int \frac{f'(x)}{f(x)}dx$$

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Angles opposite.gsp

Stressing some aspects ***and ignoring*** others (leading to what is the ***same*** and what is ***different***)

- **This can be found in the following:**
  - categorisation
    - of shapes;
    - of types of number;
  - when certain methods of calculation are appropriate and when they are not;
  - noticing when an integral is of a certain form;
  - Stressing parts of geometric drawings;
  - …etc.

Everyone is a mathematician

- ***Some 'powers' used in our early learning and used within the mathematics curriculum:***
  - Associating one thing with another;
  - Same/different;
  - Attending to order;
  - Abstracting rules from examples;
  - (this is not a definitive or complete list!)

- They don't disappear when you get older!
- You have been using them today!

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Everyone is a mathematician

- Everyone needs to have worked mathematically in order to learn their first language;
- Similar arguments apply to other learning.

- So… everyone works mathematically in so much of their learning…
- … the question is whether we can get our students to work mathematically on mathematics!

A poem

The Little Boy

by

Helen E. Buckley

Economy of education

Economy in terms of…

- the time students spend in having to learn something so that they know it well, with understanding and with confidence in their own sense of knowing;
- the effort required for them to achieve the above.

Likewise, the time and effort required from us, as teachers, in order for our students to achieve the above.

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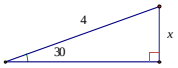
Place the following in order of difficulty

- Finding the length of side *x* in the following triangle: (NC level 4)



- Finding the general solution for sin *x* = 0.5 (i.e. x = 360n+30 or x = 360n+150) (NC level 8)

- Finding out that 2 + 2 = 4

(NC level 'exceptional performance')

Rotating dot.gsp

Whitehead, Alfred North, and Bertrand Russell (1910, 1912, 1913)
Principia Mathematica, 3 vols, Cambridge: Cambridge University Press.

352 PROLEGOMENA TO CARDINAL ARITHMETIC [PART II

***54·42.** $\vdash :: \alpha \in 2 . \supset : \beta \subset \alpha . \exists ! \beta . \beta \neq \alpha . \equiv . \beta \in \iota'' \alpha$

Dem.

$\vdash . *54·4 . \supset \vdash :: \alpha = \iota'x \cup \iota'y . \supset :$

$\beta \subset \alpha . \exists ! \beta . \equiv : \beta = \Lambda . \vee . \beta = \iota'x . \vee . \beta = \iota'y . \vee . \beta = \alpha : \exists ! \beta :$

[*24·53·56,*51·161] $\equiv : \beta = \iota'x . \vee . \beta = \iota'y . \vee . \beta = \alpha$ (1)

$\vdash . *54·25 . \text{Transp} . *52·22 . \supset \vdash : x \neq y . \supset . \iota'x \cup \iota'y \neq \iota'x . \iota'x \cup \iota'y \neq \iota'y :$

[*13·12] $\supset \vdash : \alpha = \iota'x \cup \iota'y . x \neq y . \supset . \alpha \neq \iota'x . \alpha \neq \iota'y$ (2)

$\vdash . (1) . (2) . \supset \vdash :: \alpha = \iota'x \cup \iota'y . x \neq y . \supset :.$

$\beta \subset \alpha . \exists ! \beta . \beta \neq \alpha . \equiv : \beta = \iota'x . \vee . \beta = \iota'y :$

[*51·235] $\equiv : (\exists y) . x \neq y . \beta = \iota'x :$

[*37·6] $\equiv : \beta \in \iota'' \alpha$ (3)

$\vdash . (3) . *11·11·35 . *54·101 . \supset \vdash . \text{Prop}$

***54·43.** $\vdash :. \alpha, \beta \in 1 . \supset : \alpha \cap \beta = \Lambda . \equiv . \alpha \cup \beta \in 2$

Dem.

$\vdash . *54·26 . \supset \vdash :. \alpha = \iota'x . \beta = \iota'y . \supset : \alpha \cup \beta \in 2 . \equiv . x \neq y .$

[*51·231] $\equiv . \iota'x \cap \iota'y = \Lambda .$

[*13·18] $\equiv . \alpha \cap \beta = \Lambda$ (1)

$\vdash . (1) . *11·11·35 . \supset$

$\vdash :. (\exists x, y) . \alpha = \iota'x . \beta = \iota'y . \supset : \alpha \cup \beta \in 2 . \equiv . \alpha \cap \beta = \Lambda$ (2)

$\vdash . (2) . *11·54 . *52·1 . \supset \vdash . \text{Prop}$

From this proposition it will follow, when arithmetical addition has been defined, that $1 + 1 = 2$

mathematics can be difficult

we make mathematics can be difficult

Marzana, Year 5, of about average ability, set out the following equation for her friend to solve,

$$\frac{(8x^2+10)^2-1}{2}=834$$

and then wrote out the inverse procedure to check whether she and her friend were in agreement.

$$x=\frac{\sqrt{\sqrt{2(834)+1}-10}}{8}$$

They were, but if they had not, there would have been opportunity to discuss the notation!
(Ann Boyle – excerpt from an assignment which was part of a Masters course).

For a description of the teaching method involved, see:
Hewitt, D. (1985), 'Equations', *Mathematics Teaching*, 111, pp. 15-16.
Pimm, D. (1995) *Symbols and Meaning in School Mathematics*, London: Routledge, see pp. 91-92.

The level of difficulty of any particular mathematics topic is something which is determined by people and not an inherent property of the mathematics itself

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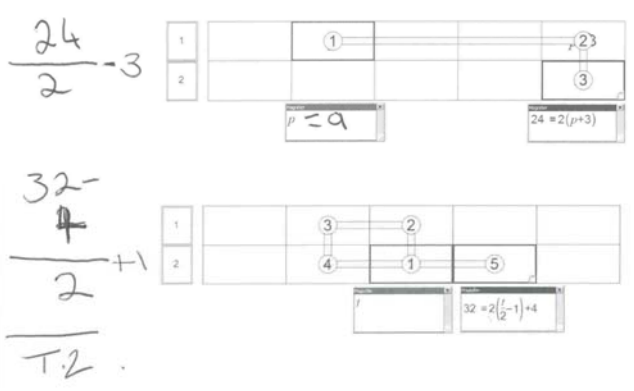
...any subject can be taught effectively in some intellectually honest form to any child at any stage of development

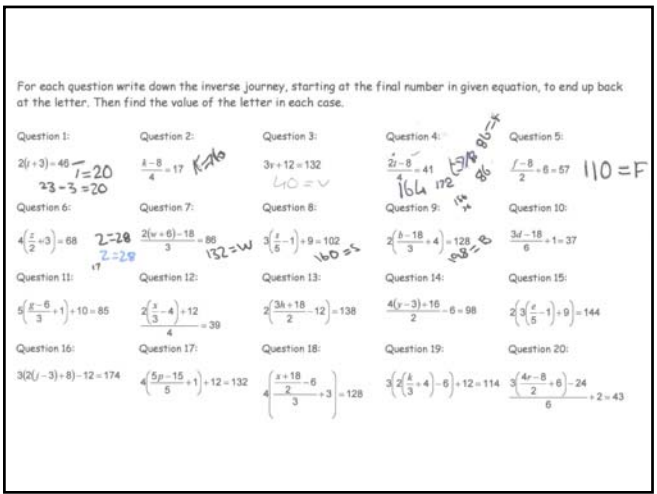
Bruner, J. S. (1960) *The process of education,* Cambridge, Massachusetts: Harvard University Press (p. 33).

Grid Algebra

Available from the Association of Teachers of Mathematics (ATM):
http://www.atm.org.uk/shop/products/sof071.html

$$\frac{2(d+3)-4}{2}-2$$





For each question write down the inverse journey, starting at the final number in given equation, to end up back at the letter. Then find the value of the letter in each case.

Question 1: $2(j+3)=46$

Question 2: $\frac{k-8}{4}=17$

Question 3: $3r+12=132$

Question 4: $\frac{2l-8}{4}=41$

Question 5: $\frac{f-8}{2}+6=57$

Question 6: $4\left(\frac{c}{2}+3\right)=68$

Question 7: $\frac{2(w+6)-18}{3}=86$

Question 8: $3\left(\frac{s}{5}-1\right)+9=102$

Question 9: $2\left(\frac{b-18}{3}+4\right)=128$

Question 10: $\frac{3d-18}{6}+1=37$

Question 11: $5\left(\frac{g-6}{3}+1\right)+10=85$

Question 12: $\frac{2\left(\frac{x}{3}-4\right)+12}{4}=39$

Question 13: $2\left(\frac{3h+18}{2}-12\right)=138$

Question 14: $\frac{4(y-3)+16}{2}-6=98$

Question 15: $2\left(3\left(\frac{e}{5}-1\right)+9\right)=144$

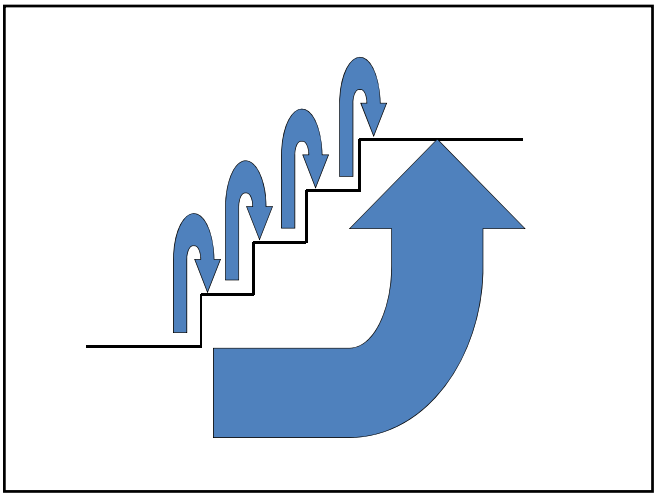
Question 16: $3(2(j-3)+8)-12=174$

Question 17: $4\left(\frac{5p-15}{5}+1\right)+12=132$

Question 18: $4\left(\frac{\frac{x+18}{2}-6}{3}+3\right)=128$

Question 19: $3\left(2\left(\frac{k}{3}+4\right)-6\right)+12=114$

Question 20: $\frac{3\left(\frac{4r-8}{2}+6\right)-24}{6}+2=43$



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Economy of education

- **Finding ways to approach apparently 'difficult' maths without reliance on previous maths knowledge. *Direct access to topics*;**

Length of garden	Number of orange tiles
3	8
4	10
5	12
6	14
7	16
8	18

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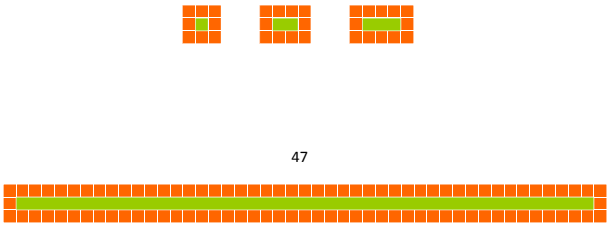

Economy of education

- Finding ways to approach apparently 'difficult' maths without reliance on previous maths knowledge. *Direct access to topics*;
- ***Stressing structure*** **helps someone see generality quicker and deeper;**

2 + 3 = 5

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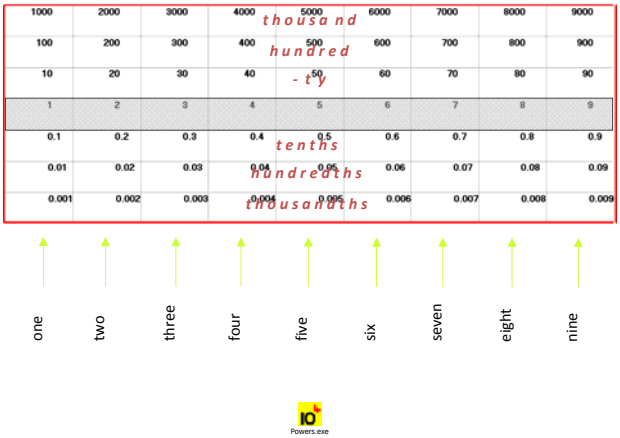
Economy of education

- Finding ways to approach apparently 'difficult' maths without reliance on previous maths knowledge. *Direct access to topics*;
- *Stressing structure* helps someone see generality quicker and deeper;
- **If I know one thing, what else comes with it? *Getting a lot from a little*;**



Economy of education

- Finding ways to approach apparently 'difficult' maths without reliance on previous maths knowledge. *Direct access to topics*;
- *Stressing structure* helps someone see generality quicker and deeper;
- If I know one thing, what else comes with it? *Getting a lot from a little*;
- ***Key images*.**

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Reflection.gsp

Economy of education

- Finding ways to approach apparently 'difficult' maths without reliance on previous maths knowledge. *Direct access to topics*;
- *Stressing structure* helps someone see generality quicker and deeper;
- If I know one thing, what else comes with it? *Getting a lot from a little;*
- *Key images*;
- ***Use of movement* can help students see generality quicker than through seeing particular examples.**

Economy of education - summary

- *Direct access*;
- *A lot from a little*;
- *Structure;*
- *Key images;*
- *Use of movement;*
- *Accessing the powers of children.*

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